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ON THE INSTABILITY OF A NONINIFORM RAREFIED PLASMA IN AN INTENSE MAGNETIC FIELD

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ON THE INSTABILITY OF A NONUNIFORM RAREFIED PLASMA IN AN INTENSE MAGNETIC FIELD

(O neustoychivosti neodnorodnoy razrezhennoy plazmy v sil'nom magnitnom pole)

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The experimental investigations on ohmic heating of a plasma by means of a current directed along an intense magnetic field [1, 2] show that instabilities develop within the plasma, which cannot be explained within the framework of ideal magnetic hydrodynamics. In references [2, 3] an interpretation of these phenomena is proposed on the basis of kinetic theory of the so-called "ionic sound" oscillation build-up by electrons transfering the current along the magnetic field. However, the appearance of such instability is only possible in an anisothermal plasma, in which the electrons are substantially hotter than the ions

$$T_e \gg T_i$$

T being the temperature.

An instability mechanism within the framework of hydrodynamics, but taking into account the conductivity's finiteness, is proposed in reference [4]. The part of this instability diminishes in conditions, when the length of electrons' free path is comparable with the characteristic dimensions of the set up.

In the following we shall examine a mechanism of instability, whose appearance is not connected with plasma's anisothermicity, and with a compulsory presence of a longitudinal electric current.

Let us admit the following allowances : 1) the pressure of the plasma is light by comparison with the magnetic pressure

$$p \gg H^2/8\pi;$$

2) the instability is developed in a shorter time than that of collision ; 3) the frequency of swaying oscillations is considerably lower than the ion cyclotron frequency

$$\ll eH/Mc,$$

and the perturbation wavelengths are considerably greater than the ion Larmor radius $\lambda \gg r_H$;
4) $H^2/8\pi \ll nMc^2$.

Let the magnetic field H be everywhere directed along the axis z, and let us admit that the magnitudes, characterizing the stationary state of the plasma, vary in the direction x. We shall examine the minor perturbations of the stationary state having the form

$$A\ (x) \exp i\ (k_z z + k_y y - \omega t).$$

Then the corrections to electron and ion distribution functions, found from the solution of the linearized kinetic equation (for electron s and ions respectively), will have the form [5]:

$$f_\alpha = -i\left(\frac{e_\alpha}{m_\alpha} E_z \frac{\partial f_{0\alpha}}{\partial v_z} + c\frac{E_y}{H_0}\frac{\partial f_{0\alpha}}{\partial x}\right)\frac{1}{\omega - k_z v_z}; \qquad (1)$$

α = i, e (ions, electrons) ; f_0 is the unperturbed distribution. Here, condition 3) was substantially used, as at its fulfilment, the particle motion across the magnetic field's lines of force is drifting :

$$\mathbf{V}_\perp = c\,[\mathbf{E}\times\mathbf{H}]/H^2.$$

Inasmuch as the examined frequencies are lower than the Larmor ion frequency, and consequently also known to be lower than the Langmuir ion frequency $(4\pi ne^2/M)^{1/2}$, plasma may be considered quasi neutral. Then, in the equation linking the variation in time of space charge density with that of the current $\mathbf{j}: \partial\rho/\partial t + \operatorname{div}\mathbf{j} = 0,$ we may drop the first term

$$\operatorname{div}\mathbf{j} = 0. \qquad 2)$$

The perturbation of the current's density j is expressed in the following manner : the longitudinal component has by definition the form

$$j_z = e_a \int v \, f_a \, do;$$

as to the transverse component in case of an intense magnetic field and low frequencies (condition 3), it may be directly expressed through the electric field with the help of the so-called "static" dielectric constant $\varepsilon_\perp = 4\pi n M c^2/H^2$:

$$\mathbf{j}_\perp = \frac{\varepsilon_\perp}{4\pi} \frac{\partial \mathbf{E}_\perp}{\partial t} = -\frac{i\omega}{4\pi} \varepsilon_\perp \mathbf{E}_\perp. \qquad (3)$$

Finally, still another simplification may be introduced in the problem of interest to us : to consider the electric field as potential $E = -\vec{\nabla}\varphi$. This is valid when the inequality $\mathbf{E} \gg \frac{1}{c} \omega \mathbf{A}$ is fulfilled, or, expressing A by means of current's j density from equation $\Delta \mathbf{A} = \frac{4\pi}{c} \mathbf{j}$),

$$\mathbf{E} \gg \frac{4\pi}{c^2} \frac{\bar{\mathbf{j}}}{k^2}. \qquad (4)$$

For transverse components, and using relation (3), the inequality (4) may be reduced to the following :

$$\frac{\omega^2}{k^2} \ll \frac{H^2}{4\pi n M}.$$

This means that we are restricted to perturbations propagating with phase velocities considerably lower than the Alfven velocity. According to the estimate, in whose deduction we will not indulge on account of it being too cumbersome, the longitudinal component is reduced to the condition

$$\frac{\omega^2}{k_z^2} \ll \frac{H^2}{4\pi n M}.$$

Now, utilizing equations (1) — (3), and the condition of electric field's potentiality, we shall obtain after simple computations the following equation for scalar potential φ perturbation

$$\frac{d^2\varphi}{dx^2} - F(\omega, k, x,) \varphi = 0, \qquad (5)$$

$$F(\omega, k, x) = \frac{\omega_H^2}{\omega} k_z^2 \int \frac{v_z dv_z}{\omega - k_z v_z} \left\{ \left[\frac{M}{m} \frac{\partial f_{0e}}{\partial v_z} + \frac{\partial f_{0i}}{\partial v_z} \right] + \frac{k_y}{k_z} \frac{1}{\omega_H} \frac{\partial}{\partial x} (f_{0i} - f_{0e}) \right\} \tag{5}$$

$$\left(\omega_H = \frac{eH}{Mc} \right).$$

(When taking integrals the integrand poles in this expression must be by-passed underneath).

In an homogenous plasma, (5) provides the known dispersion equation of ionic sound oscillations

$$\int \frac{v_z dv_z}{\omega - k_z v_z} \left[\frac{M}{m} \frac{\partial f_{0e}}{\partial v_z} + \frac{\partial f_{0i}}{\partial v_z} \right] = 0.$$

As to the nonuniform plasma, the solutions of equation (5) for φ decreasing both ways (with $x \to \pm\infty$) must be searched for. Together with this requirement, equation (5) determines the proper values .

Local solutions near the point x, where

$$F(\omega, k, x) = 0. \tag{6}$$

are solutions of such a type.

In the neighborhood of such x, equation (5) takes, generally speaking, the form of Eyre equation from a complex argument (since ω is a complex magnitude). Such equation has solutions, attenuating both ways from the point where $F(\omega, k, x) = 0$. (This point is equivalent to the "turning point" in the Schrödinger equation for the one-dimensional motion).

In the equation (6), playing the part of a "dispersion equation" in a nonuniform plasma, the term with $\frac{\partial}{\partial x}(f_{0i} - f_{0e})$ may become effective even small nonuniformity, provided $k_y \gg k_z$.

The application of the described deductions to the case, when the particle distribution by velocities is Maxwellian

$$f_{0\alpha} = \frac{1}{\sqrt{2\pi T / m_\alpha}} e^{-m_\alpha v_z^2/2T} \quad (T_i = T_e = T)$$

(n and T being functions of coordinate x), leads to the following results :

1. If $n(x) = \text{const.}$, $T(x) \neq \text{const.}$, then for frequencies in the interval $\sqrt{\frac{T}{M}} \ll \frac{\omega}{k} \ll \sqrt{\frac{T}{m}}$

and under the condition

$$\frac{k_y^2}{k_z^2} \left(\frac{d \ln T}{dx} \right)^2 r_H^2 \gg 1$$

equation (4) will take the form

$$\omega^3 + \frac{k_y}{k_z}\frac{d\ln T}{dx} k_z^3 \frac{2T^2}{M^2\omega_H} = 0. \tag{7}$$

Here, radical

$$\omega = \frac{1}{\sqrt{3}}(1 + 2i)\left(\frac{k_y}{k_z}\frac{d\ln T}{dx} k_z^3 \frac{2T^2}{M^2\omega_H}\right)^{1/3},$$

providing the instability is always present. Let us remark, that the restriction to the frequency interval

$$\sqrt{\frac{T}{M}} \ll \frac{\omega}{k} \ll \sqrt{\frac{T}{m}},$$

is, as is well known from the theory of ionic oscillations of a uniform plasma, equivalent to the hydrodymaic approximation in which the adiabat index for an electron gas is equal to the unity. It may be shown that the results of this point are included in the hydrodynamic approach for a nonuniform plasma too. Accordingly, the equation (7) does not all the radicals of the dispersion equation (6)

2. If $T(x) = \text{const}$, $n(x) \neq \text{const.}$, the examination of equation (6) shows, that the available complex radicals correspond to the damping of oscillations (stable plasma).

The instability at variable temperature may be interpreted in the following descriptive fashion: In a homogeneous plasma, the "ionic electrostatic wave" represents oscillations (sound), propagating along the lines of force of the magnetic field H. In a nonuniform plasma ($T(x) \neq \text{const}$) in a "slanting" wave (k_z, $k_y \neq 0$), the transverse motion with the velocity $[\mathbf{E}\times\dot{\mathbf{H}}]/H^2$ leads to heat transfer. A direction of wave propagation, sign $\frac{k_y}{k_z}$, may be chosen in such a manner, that a continous heat inflow take place in the phase of plasma "thickening" in the wave, when temperature increases, from the region with a greater unperturbed temperature. This would precisely constitute the cause leading to oscillation accretion.

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